

On elimination of the Gribov ambiguity

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Abstract

We find a strong coupling expansion around the non-trivial extremum of the Yang-Mills action. It is shown that the developed formalism is the Gribov ambiguity free since each order of the developed perturbation theory is transparently gauge invariant. The success is a consequence of the restriction: calculations are not going beyond the norm of the S -matrix element.

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1 Introduction

In the late seventieth V.Gribov discovered that it was impossible to extract unambiguously the non-Abelian gauge symmetry degrees of freedom if the gauge field was strong with the Coulomb gauge to be applied [10]. It was shown later that the same phenomenon appeared in the arbitrary gauge fixing conditions [5, 28]. On the other hand, the canonical quantization scheme certainly prescribes to extract the symmetry degrees of freedom [6].

This problem is unavoidable, Sec.2.2, in the general approach to the Yang-Mills theory, but it can be resolved calculating the norm of the amplitude, $|\mathbb{Z}|$, Sec.4.4.

Formally, we present a partial solution of the problem since the phase of \mathbb{Z} is excluded from consideration. Then the result of Gribov, Atiyah and Singer [10, 5, 28] might signify that the problem is unsolvable in the presence of the phase. In other words, we will argue that the Yang-Mills field theory is free from ambiguities if it is used for the description of observables: in this sense it can exist and that will do. The application of this formalism restricted by the norm was deduced in a number of papers, see e.g. [21, 24].

1.1 Solution of the Gribov problem in brief

It is necessary to give a more exact answer to the question: what does the arbitrariness of the phase mean? The optical theorem:

$$i\mathbb{Z}\mathbb{Z}^\dagger = (\mathbb{Z} - \mathbb{Z}^\dagger), \quad (1.1)$$

will be involved for this purpose [22]. Therefore, the wish to leave the phase arbitrary means that only the absorption part of amplitudes, $\Delta\mathbb{Z} = (\mathbb{Z} - \mathbb{Z}^\dagger)/2i$, would be the object of our calculations.

The functional integral representation of $\Delta\mathbb{Z}$ is unknown. For this reason we will start with \mathbb{Z} and find the functional integral representation for $\Delta\mathbb{Z}$ using (1.1).

It was shown that $\Delta\mathbb{Z}$ is defined on the δ -like (Dirac) functional measure [20]:

$$DM(A) = \prod_x dA_{a\mu}(x) \delta \left(\frac{\delta S(A)}{\delta A^{a\mu}(x)} + \hbar J_{a\mu}(x) \right), \quad (1.2)$$

where $A_{a\mu}$ is the Yang-Mills potential, a is the colour index. The reason of Dirac measures appearance is the cancellation of contributions in the difference of r.h.s. of (1.1). They are "unnecessary" from the point of view of conservation of total probability. The boundary condition that the end points of action do not vary, (2.16), has been used. Sec.2. contains the derivation of (1.2) for the Yang-Mills theory.

We have found that the corresponding quantum perturbations are excited by the Gauss operator $\exp\{-i\mathbb{K}(J)\}$ in the vicinity of $J_{a\mu} = 0$, see Eq.(2.22). It can be shown [22] that the theory restores in full measure the "weak-coupling" expansion of the type described in the papers [15, 16, 8].

The "correspondence principle" written in (1.2) is strict, it does not depend on the value of Plank constant \hbar . Thus, it defines the rule how the quantum force, $J_{a\mu}(x)$, must be transformed under the transformation of $A_{a\mu}(x)$. The latter is impossible in the general functional integral representation of \mathbb{Z} [7, 26], see also [29].

The Dirac measure orders to perform the transformation in the class of strict solutions, $u_{a\mu}(x)$, of the sourceless (with $J_{a\mu} = 0$) Lagrange equation. This stands for [11, 14, 17, 18, 19] mapping into the coset space \mathcal{G}/\mathcal{H} :

$$u_{a\mu} : A_{a\mu}(x) \rightarrow \{\lambda\} \in \mathcal{G}/\mathcal{H}, \quad (1. 3)$$

where \mathcal{G} is the symmetry group of the problem and \mathcal{H} is the invariance group of $u_{a\mu}$. The qualitative reason of this choice is the following: after having got the ground state field, $\forall u_{a\mu}(x)$, the freedom in the choice of the value of integration constants, $\{\lambda\}$, is what remains from the continuum of the field degrees of freedom. The gauge phases $\{\Lambda^a\} \subset \{\lambda\}$.

The mapping (1. 3) involves the splitting [22]:

$$J_{a\mu}(\mathbf{x}, t) \rightarrow (j_\xi, j_\eta)(t), \quad \{\xi, \eta\} \in T^*W, \quad (1. 4)$$

where⁴ the symplectic subspace $T^*W \subseteq W$ and W is the physical coset space. The definition of the physical coset space is given in Sec.2.2.

The transformed perturbations generating operator, $\exp\{-i\mathbf{k}(j)\}$, is still Gaussian. It acts in T^*W and generates the "strong-coupling" perturbation series if $u_{a\mu} \sim 1/\sqrt{g}$, g is the interaction constant. The question of the existence of perturbation series of such a type presents a separate problem. We will assume that the series exist. The expansion of the operator exponent $\exp\{-i\mathbf{k}(j)\}$ produces also the asymptotic series over the non-negative even powers of \hbar [25].

It will be shown that $\{\Lambda^a(\mathbf{x}, t)\} \not\subset \{\xi, \eta\}$. Therefore, each order of the expansion of $\exp\{-i\mathbf{k}(j)\}$ is the transparently gauge invariant quantity and no gauge fixing procedure will be required. This is the main result. The preliminary verse of it was given in [23].

1.2 Agenda

We will draw attention to the following two questions.

We will find that

$$W = T^*W + R, \quad (1. 5)$$

where $\{\Lambda^a\} \in R$ since there is no conjugate to Λ^a gauge charge dependence in the field $u_{a\mu}$.

The mapping (1. 3) is singular since $\dim T^*W < \infty$. We will show that the singularity can be isolated and cancelled by the normalization. This result allows to conclude that no gauge fixing problem will arise because of the renormalization procedure.

The structure of the paper is given in the table of Contents.

2 Perturbation theory

We will consider the theory with the action:

$$S(A) = -\frac{1}{4} \int d^4x F_{\mu\nu}^a(A) F_a^{\mu\nu}(A) \quad (2. 6)$$

⁴The developed formalism will not be manifestly Lorentz covariant. The space component \mathbf{x} and time t , $x^2 = t^2 - \mathbf{x}^2$, were decoupled for this reason.

The Yang-Mills fields

$$F_{a\mu\nu}(A) = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - gC_a^{bc} A_{b\mu} A_{c\nu} \quad (2.7)$$

are the covariants of the non-Abelian gauge transformations. The group will not be specified. The matrix notation: $A_{a\mu}\omega_a = A_\mu$ will be also used.

We will calculate the quantity⁵:

$$\mathcal{N} = |\mathcal{Z}|^2, \quad (2.8)$$

where

$$\mathcal{Z} = \int DA e^{iS_C(A)}, \quad DA = \prod_{\mathbf{x}, t \in C} \prod_{a, \mu} \frac{dA_{a, \mu}(\mathbf{x}, t)}{\sqrt{2\pi}}, \quad (2.9)$$

is defined on the Minkowski metric. The Mills complex time formalism will be used to avoid the light cone singularities [27]. For example,

$$C : t \rightarrow t + i\varepsilon, \quad \varepsilon \rightarrow +0, \quad -\infty \leq t \leq +\infty. \quad (2.10)$$

At the very end one must take $\varepsilon = +0$. The Mills formalism restores the Feynman's $i\varepsilon$ -prescription.

2.1 Dirac measure

The double integral:

$$\mathcal{N} = \int DA^+ DA^- e^{iS_C(A^+) - iS_C^*(A^-)} \quad (2.11)$$

will be calculated using the equality (1.1). To extract the Dirac measure⁶, one must introduce the mean trajectory, $A_{a\mu}$, and the virtual deviation, $a_{a\mu}$, instead of A_μ^\pm :

$$A_{a\mu}^\pm(x) = A_{a\mu}(x) \pm a_{a\mu}(x). \quad (2.12)$$

It will be shown that the matrix a_μ is the covariant of gauge transformations:

$$a_\mu \rightarrow a'_\mu = \Omega a_\mu \Omega^{-1} \quad (2.13)$$

The transformation (2.12) is linear and the differential measure

$$DA^+ DA^- = \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} dA_{a\mu}(x) \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} \frac{da_{a\mu}(x)}{\pi} \equiv DADa. \quad (2.14)$$

is defined on the complete time contour $C + C^*$. Notice that

$$A_{a\mu}(\mathbf{x}, t \in C^*) = A_{a\mu}^*(\mathbf{x}, t \in C). \quad (2.15)$$

The "closed-path" boundary conditions:

$$a_{a\mu}(x \in \sigma_\infty) = 0, \quad (2.16)$$

⁵The generalization was considered in [24, 25].

⁶The term "δ-like (Dirac) measure" have been taken from [9].

where σ_∞ is the remote time-like hypersurface, is assumed. We will demand that the surface terms are cancelled in the difference $S_C(A^+) - S_C^*(A^-)$, i.e.

$$\int dx \partial_\mu (A_{a\nu} \partial^\mu A^{a\nu})^+ = \int dx \partial_\mu (A_{a\nu} \partial^\mu A^{a\nu})^-. \quad (2.17)$$

Therefore, not only the trivial pure gauge fields can be considered on σ_∞ .

Expanding $S(A \pm a)$ over $a_{a\mu}$, one can write:

$$S(A + a) - S^*(A - a) = U(A, a) + 2\text{Re} \int_C dx a_{a\mu}(x) \frac{\delta S(A)}{\delta A_{a\mu}(x)}. \quad (2.18)$$

This equality will be used as the definition of the remainder term, $U(A, a)$. With the ε accuracy, $U(A, a) = O(a^3)$, i.e $U(A, a)$ introduces the interactions. Notice that

$$\frac{\delta S(A)}{\delta A^{a\mu}(x)} = D_a^{\nu b} F_{\mu\nu}^b \quad (2.19)$$

is the covariant of gauge transformations. Therefore, the remainder term, $U(A, a)$, is the gauge invariant if (2.13) is held.

Inserting (2.14) and (2.18) into (2.11), we find:

$$\mathcal{N} = \int DA \int Da e^{2i\text{Re} \int_C dx a_a^\mu(x) D_a^{\nu b} F_{\mu\nu}^b} e^{iU(A, a)}. \quad (2.20)$$

The integrals over $a_{a\mu}(x)$ will be calculated perturbatively. For this purpose one can use the identity:

$$e^{iU(A, a)} = \lim_{\zeta_{a\mu} = J_{a\mu} = 0} e^{-i\mathbb{K}(J, \zeta)} e^{2i\text{Re} \int_C dx a_{a\mu}(x) J^{a\mu}(x)} e^{iU(A, \zeta)}, \quad (2.21)$$

where

$$2\mathbb{K}(J, \zeta) = \text{Re} \int_C dx \frac{\delta}{\delta J^{a\mu}(x)} \frac{\delta}{\delta \zeta_{a\mu}(x)}. \quad (2.22)$$

In the future we will omit the sign of the limit bearing in mind the prescription: the auxiliary variables, $J_{a\mu}$ and $\zeta_{a\mu}$, must be taken equal to zero at the very end of calculations.

Assuming that the perturbation series will exist, the insertion of the Eq.(2.21) into (2.20) gives the desired expression:

$$\mathcal{N} = e^{-i\mathbb{K}(J, \zeta)} \int DM(A) e^{iU(A, \zeta)}, \quad (2.23)$$

where

$$\begin{aligned} DM(A) &= \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} dA_{a\mu}(x) \int \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} \frac{da_{a\mu}(x)}{\pi} e^{2i\text{Re} \int_C dx a^{a\mu}(x) (D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu}(x))} = \\ &= \prod_{\mathbf{x}, t \in C+C^*} \prod_{a, \mu} dA_{a\mu}(x) \delta(D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu}(x)) \end{aligned} \quad (2.24)$$

is the functional Dirac measure. The functional δ -function on the complex time contour $C + C^*$ has the definition:

$$\prod_{\mathbf{x}, t \in C + C^*} \delta(D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu}) = \prod_{\mathbf{x}, t \in C} \delta(\text{Re}(D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu})) \delta(i\text{Im}(D_a^{\nu b} F_{\mu\nu}^b - J_{a\mu})) \quad (2. 25)$$

where the equality (2. 15) was used.

It can be shown that (2. 23) gives the ordinary perturbation theory (pQCD) [22, 15, 16, 8] if the equation:

$$D_{ab}^\nu F_{\mu\nu}^b = J_{a\mu} \quad (2. 26)$$

is expanded in the vicinity of $A_{a\mu} = 0$. Notice that the Eq.(2. 26) is not gauge covariant because of $J_{a\mu}(x)$.

2.2 Definition of physical coset space

The approach based only on the Dirac measure, DM , is incomplete since all strict solutions $u_{a\mu}^i$ of the equation (2. 26) must be taken into consideration:

$$\mathcal{N} = \sum_i \mathcal{N}_i, \quad (2. 27)$$

where \mathcal{N}_i corresponds to $u_{a\mu}^i$. It is necessary to extract one, physical, term in (2. 27) since each i -th solution belongs to the different symmetry class [30].

Notice that the non-diagonal terms,

$$Z_i Z_j^* + Z_i^* Z_j, \quad i \neq j, \quad (2. 28)$$

are absent in the sum (2. 27) as the consequence of orthogonality of the Hilbert spaces, see (2. 24). This allows to offer the following selection rule.

Corollary. *If there are no special external conditions and if the coset space W_i is adjusted to $u_{a\mu}^i$ then the dimension of the physical coset space, $\dim W$, is defined by the condition:*

$$\dim W = \max\{\dim W_i\}. \quad (2. 29)$$

The solution of (2. 26) must be chosen in accordance with this selection rule.

Indeed, having (2. 27) and noting the absence of the non-diagonal terms of (2. 28) type, one may use the notion of the "situation of general position" ordinary for classical mechanics [4, 1]. It means the absence of special boundary conditions to the Eq. (2. 26). Then in the sum over the strict solutions of the Lagrange equation, namely, the one which gives the largest contribution, $u_{a\mu}$, must be left. Other contributions would be realized on the zero measure since the (2. 27) includes summation over initial conditions⁷, see Sec.4.3.

Following the selection rule (2. 29) the Gribov ambiguity actually presents the problem in the non-Abelian gauge theory since we know, at least, the $O(4) \times O(2)$ -invariant

⁷For instance, the trivial trajectory corresponds to the particle lying motionless in the semiclassical approximation at the bottom of a potential hole. This type of trajectories belongs to a definite topology class [30] and must be refused as it follows from our selection rule, Corollary.

strict solution of the $SU(2)$ Yang-Mills equation. The corresponding coset space has $\dim W = 8$ plus the (infinite) gauge groups dimension. Therefore, following the selection rule (2. 29), the excitations in the vicinity of $A_{a\mu} = 0$ are realized on the zero measure.

The *ansatz* [12, 31]:

$$\sqrt{g}u_\mu^a = \eta_{\mu\nu}^a \partial^\nu \ln u \quad (2. 30)$$

for the $SU(2)$ Yang-Mills potential u_μ^a leads to the conformal scalar field theory, see e.g. [2]. For our purpose it is enough to know the existence of the exact solution [3, 2]:

$$u(\mathbf{x}, t) = \left\{ \frac{(\gamma - \gamma^*)^2}{(x - \gamma)^2(x - \gamma^*)^2} \right\}^{1/2}, \quad (2. 31)$$

where

$$\gamma_\mu = \xi_\mu + i\eta_\mu, \quad \gamma_\mu \gamma^\mu = g^{\mu\nu} \gamma_\mu \gamma_\nu = \gamma_0^2 - \gamma_i^2, \quad i = 1, 2, 3 \quad (2. 32)$$

and ξ_μ and η_μ are the real numbers. Their physical domain is defined by inequalities:

$$-\infty \leq \xi_\mu \leq +\infty, \quad -\infty \leq \gamma_i \leq +\infty, \quad \sqrt{\eta_\mu \eta^\mu} = \eta \geq 0. \quad (2. 33)$$

The latter condition means that the energy of $u_{a\mu}$ is nonnegative.

The solution (2. 31) is regular in the Minkowski metric for $\eta > 0$ and has the light-cone singularity at $\eta = 0$, i.e the solution is singular at the border $\partial_\eta W$. We will regularize it continuing contributions on the Mills complex-time contour, Sec.2.1. The solution (2. 31) has the finite energy and no topological charge. There also exist its elliptic generalizations [2]

The selection rule (2. 29) defines the ground state of the theory on the Dirac measure. Thus, a chosen vacuum optionally has the lowest energy. This definition of the ground state is useful if the finite-time dynamic problem is considered.

3 Mapping into the coset space

Considering the general transformation:

$$(A_{a\mu}, P_{a\mu})(\mathbf{x}, t) \rightarrow (\lambda, \kappa)_\alpha(t), \quad (3. 1)$$

where $A_{a\mu}$ is the arbitrary set of fields and $P_{a\mu}$ is the conjugate momentum, one must conserve the dimension of the path integral measure:

$$\dim DM(A_{a\mu}, \pi) = \dim DM(\lambda, \kappa), \quad (3. 2)$$

where

$$DM(\lambda, \kappa) = \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t).$$

Therefore, the Eq.(3. 2) defines the set $\{\alpha\}$. For the Yang-Mills theory the set $(\lambda, \kappa)_\alpha$ includes the gauge phase $\Lambda_a(\mathbf{x}, t)$. For a more confidence, one may consider the theory on the space lattice.

It is assumed that the time dependent variables will be defined on the complete Mills time contour, $C + C^*$. We will formulate the general method of mapping (3. 1) into the infinite dimensional phase space Γ_∞ , Sec.3.2, and then will find the reduction procedure, $\Gamma_\infty \mapsto W$ on the second stage of the calculation, Sec.4.1.

3.1 First order formalism

The action in terms of the electric field, $E_a^i = F_a^{i0}$, $i = 1, 2, 3$, looks as follows:

$$S(\mathbf{A}, \mathbf{E}) = \int dx \left\{ \dot{\mathbf{A}}_a \mathcal{D}\mathbf{E}_a + \frac{1}{2}(\mathbf{E}_a^2 + \mathbf{B}_a^2) - A_{0a}(\mathcal{D}\mathbf{E})_a \right\}, \quad (3.3)$$

where the magnetic field $\mathbf{B}_a(\mathbf{A}) = \text{rot}\mathbf{A}_a + \frac{1}{2}(\mathbf{A} \times \mathbf{A})_a$. The corresponding Dirac measure is:

$$DM(\mathbf{A}, \mathbf{P}) = \prod_a \prod_x d\mathbf{A}_a(x) d\mathbf{P}_a(x) \delta(\mathcal{D}\mathbf{P}_b) \times \\ \times \delta\left(\dot{\mathbf{P}}_a(x) + \frac{\delta H_J(\mathbf{A}, \mathbf{P})}{\delta \mathbf{A}_a(x)}\right) \delta\left(\dot{\mathbf{A}}_a(x) - \frac{\delta H_J(\mathbf{A}, \mathbf{P})}{\delta \mathbf{P}_a(x)}\right), \quad (3.4)$$

where $d\mathbf{A}_a(x)d\mathbf{P}_a(x) = \prod_i dA_{ia}dP_{ai}(x)$, $i = 1, 2, 3$, and the total Hamiltonian

$$H_J = \frac{1}{2} \int d^3x (\mathbf{P}_a^2 + \mathbf{B}_a^2) + \int d^3x \mathbf{j}_a \mathbf{A}_a. \quad (3.5)$$

Notice that the dependence on A_{a0} was integrated out and the Gauss law, $\mathcal{D}_b^a \mathbf{P}_b = 0$, was appeared as a result in (3.4). The Faddeev-Popov *ansatz* was not used for the definition of the integral over $A_{a\mu}$. The perturbations generating operator \mathbb{K} and the remainder potential term U stay unchanged, see (2.22) and (2.18).

The integrals with the measure (3.4) will be calculated using new variables.

3.2 General mechanism of transformations

Proposition 1. *The Jacobian of transformation of the Dirac measure is equal to one [22].*

One can insert the unite

$$1 = \frac{1}{\Delta(\lambda, \kappa)} \int \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t) \times \\ \times \prod_{a, \mathbf{x}} \delta(\mathbf{A}_a(\mathbf{x}, t) - \mathbf{u}_a(\mathbf{x}; \lambda(t), \kappa(t))) \delta(\mathbf{P}_a(\mathbf{x}, t) - \mathbf{p}_a(\mathbf{x}; \lambda(t), \kappa(t))), \quad (3.6)$$

into the integral (2.23) and integrate over \mathbf{A}_a and \mathbf{P}_a using the δ -functions of (3.6). In this case the transformation is performed. Otherwise, if the δ -functions of (3.4) are used, \mathbf{u}_a and \mathbf{p}_a will play the role of constraints and (3.6) will present the Faddeev-Popov *ansatz*. It must be noted that the both ways of calculation must lead to the identical ultimate result because of the δ -likeness of measures in (3.4) and (3.6). The first way is preferable since it does not imply the ambiguous gauge fixing procedure [10, 5, 28].

The arbitrary given composite functions $\mathbf{u}_a(\mathbf{x}; \lambda(t), \kappa(t))$ and $\mathbf{p}_a(\mathbf{x}; \lambda(t), \kappa(t))$ must obey the condition:

$$\Delta(\lambda, \kappa) = \int \prod_{\alpha, t} d\lambda'_\alpha(t) d\kappa'_\alpha(t) \prod_{a, \mathbf{x}} \delta(\lambda' \mathbf{u}_{a, \lambda} + \kappa' \mathbf{u}_{a, \kappa}) \delta(\lambda' \mathbf{p}_{a, \lambda} + \kappa' \mathbf{p}_{a, \kappa}) \neq 0, \quad (3.7)$$

where

$$\mathbf{u}_{a,X} \equiv \frac{\partial \mathbf{u}_a}{\partial X}, \quad \mathbf{p}_{a,X} \equiv \frac{\partial \mathbf{p}_a}{\partial X}, \quad X = (\lambda_\alpha, \kappa_\alpha).$$

The summation over the repeated index, α , will be assumed.

The transformed measure:

$$DM(\lambda, \kappa) = \frac{1}{\Delta(\lambda, \kappa)} \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t) \times \\ \times \prod_{a, \mathbf{x}} \delta \left(\dot{\lambda} \mathbf{u}_{a, \lambda} + \dot{\kappa} \mathbf{u}_{a, \kappa} - \frac{\delta H_J(\mathbf{u}, \mathbf{p})}{\delta \mathbf{p}_a} \right) d \left(\dot{\lambda} \mathbf{p}_{a, \lambda} + \dot{\kappa} \mathbf{p}_{a, \kappa} + \frac{\delta H_J(\mathbf{u}, \mathbf{p})}{\delta \mathbf{u}_a} \right)$$

can be diagonalized introducing the auxiliary function(al) h_J :

$$DM(\lambda, \kappa) = \frac{1}{\Delta(\lambda, \kappa)} \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t) \int \prod_{\alpha, t} d\lambda'_\alpha(t) d\kappa'_\alpha(t) \times \\ \times \delta \left(\lambda'_\alpha - \left(\dot{\lambda}_\alpha - \frac{\delta h_J(\lambda, \kappa)}{\delta \kappa_\alpha} \right) \right) \delta \left(\kappa'_\alpha - \left(\dot{\kappa}_\alpha + \frac{\delta h_J(\lambda, \kappa)}{\delta \lambda_\alpha} \right) \right) \times \\ \times \prod_{a, \mathbf{x}} \delta \left(\mathbf{u}_{a, \lambda} \lambda' + \mathbf{u}_{a, \kappa} \kappa' + \{\mathbf{u}, h_J\}_a - \frac{\delta H_J}{\delta \mathbf{p}_a} \right) \delta \left(\mathbf{p}_{a, \lambda} \lambda' + \mathbf{p}_{a, \kappa} \kappa' + \{\mathbf{p}, h_J\}_a + \frac{\delta H_J}{\delta \mathbf{u}_a} \right), \quad (3. 8)$$

where $\{, \}$ is the Poisson bracket.

Let us assume now that \mathbf{u}_a , \mathbf{p}_a and h_J are chosen in such a way that:

$$\{\mathbf{u}_a, h_J\} - \frac{\delta H_J}{\delta \mathbf{p}_a} = 0, \quad \{\mathbf{p}_a, h_J\} + \frac{\delta H_J}{\delta \mathbf{u}_a} = 0. \quad (3. 9)$$

Then, having the condition (3. 7), the transformed measure takes the form, see (3. 8):

$$DM(\lambda, \kappa) = \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t) \delta \left(\dot{\lambda}_\alpha - \frac{\delta h_J(\lambda, \kappa)}{\delta \kappa_\alpha} \right) \delta \left(\dot{\kappa}_\alpha + \frac{\delta h_J(\lambda, \kappa)}{\delta \lambda_\alpha} \right), \quad (3. 10)$$

where the functional determinant $\Delta(\lambda, \kappa)$ was cancelled.

As a result,

$$\mathcal{N} = e^{-i\mathbb{K}(J, \zeta)} \int DM(\lambda, \kappa) e^{iU(\mathbf{u}, \zeta)} \quad (3. 11)$$

where $\mathbb{K}(J, \zeta)$ was defined in (2. 22), $DM(\lambda, \kappa)$ was defined in (3. 10) and $U(\mathbf{u}, \zeta)$ was introduced in (2. 18). Therefore, the Jacobian of transformation is equal to one, i.e. in the frame of the conditions (3. 7) and (3. 21) the phase space volume is conserved. Q.E.D.

We will consider the case when h_J is the linear over $J(\mathbf{x}, t)$ functional:

$$h_J(\lambda, \kappa) = h(\lambda, \kappa) + \int d\mathbf{x} \mathbf{J}_a(\mathbf{x}, t) \mathbf{Y}_a(\mathbf{x}; \lambda, \kappa), \quad (3. 12)$$

where \mathbf{Y}_a are the arbitrary vector functions. Transforming the theory we get to the dynamical problem for $\lambda_\alpha(t)$ and $\kappa_\alpha(t)$:

$$\begin{aligned}\dot{\lambda}_\alpha &= \frac{\delta h_J(\lambda, \kappa)}{\delta \kappa_\alpha} = \frac{\delta h(\lambda, \kappa)}{\delta \kappa_\alpha} + \int d\mathbf{x} \mathbf{J}_a \frac{\delta \mathbf{Y}_a}{\delta \kappa_\alpha} \equiv h_{\kappa_\alpha} + \int d\mathbf{x} \mathbf{J}_a \mathbf{Y}_{a,\kappa}, \\ \dot{\kappa}_\alpha &= -\frac{\delta h_J(\lambda, \kappa)}{\delta \lambda_\alpha} = -\frac{\delta h(\lambda, \kappa)}{\delta \lambda_\alpha} - \int d\mathbf{x} \mathbf{J}_a \frac{\delta \mathbf{Y}_a}{\delta \lambda_\alpha} \equiv -h_{\lambda_\alpha} - \int d\mathbf{x} \mathbf{J}_a \mathbf{Y}_{a,\lambda}.\end{aligned}\quad (3.13)$$

The equality (3.12) was used here.

Proposition 2. *If (3.12) is held then the transformation (3.1) induces the splitting:*

$$\mathbf{J}_a \rightarrow \{j_\lambda, j_\kappa\} \quad (3.14)$$

The proof of the splitting comes from the identity:

$$\begin{aligned}& \prod_{\alpha,t} \delta \left(\dot{\lambda}_\alpha - \frac{\delta h_J(\lambda, \kappa)}{\delta \kappa_\alpha} \right) \delta \left(\dot{\kappa}_\alpha + \frac{\delta h_J(\lambda, \kappa)}{\delta \lambda_\alpha} \right) = \\ &= \exp\{-i\mathbf{k}(j, e)\} \exp \left\{ 2i \text{Re} \int_C d\mathbf{x} dt \mathbf{J}_a(\mathbf{x}, t) (e_\lambda \mathbf{Y}_{a,\kappa} + e_\kappa \mathbf{Y}_{a,\lambda}) \right\} \times \\ & \times \prod_{\alpha,t} \delta(\dot{\lambda}_\alpha - h_{\kappa_\alpha} - j_{\lambda_\alpha}) \delta(\dot{\kappa}_\alpha + h_{\lambda_\alpha} + j_{\kappa_\alpha}),\end{aligned}\quad (3.15)$$

where

$$2\mathbf{k}(j, e) = \text{Re} \int_C dt \left(\frac{\delta}{\delta j_\lambda} \frac{\delta}{\delta e_\lambda} + \frac{\delta}{\delta j_\kappa} \frac{\delta}{\delta e_\kappa} \right). \quad (3.16)$$

At the very end one must take $j_X = e_X = 0$, $X = (\lambda, \kappa)$. The equality (3.15) can be derived using the functional δ -functions Fourier transformation (2.24).

Inserting (3.15) into (3.11) we find the completely transformed representation for \mathcal{N} , where the individual to each degree of freedom quantum sources, j_X , $X = (\lambda, \kappa)$, appears. The transformed representation of \mathcal{N} looks like:

$$\mathcal{N} = e^{-i\mathbf{k}(j,e)} \int DM(\lambda, \kappa) e^{iU(\mathbf{u}, \mathbf{e})}, \quad (3.17)$$

where

$$DM(\lambda, \kappa) = \prod_{\alpha,t} d\lambda_\alpha(t) d\kappa_\alpha(t) \delta(\dot{\lambda} - h_\kappa(\lambda, \kappa) - j_\lambda) \delta(\dot{\kappa} + h_\lambda(\lambda, \kappa) + j_\kappa), \quad (3.18)$$

$$\mathbf{e}_a = e_\lambda \mathbf{Y}_{a,\kappa} + e_\kappa \mathbf{Y}_{a,\lambda} \quad (3.19)$$

and $\mathbf{k}(j, e)$ was defined in (3.16).

Proposition 3. *The Eq.(3.9) and the measure (3.18) define the classical flow for arbitrary $h_J(\lambda, \kappa)$.*

Indeed,

$$\dot{\mathbf{u}}_a = \dot{\lambda} \mathbf{u}_{a,\lambda} + \dot{\kappa} \mathbf{u}_{a,\kappa} = \{\mathbf{u}_a, h_J\} = \frac{\delta H_J}{\delta \mathbf{P}_a},$$

$$\dot{\mathbf{p}}_a = \dot{\lambda} \mathbf{p}_{a,\lambda} + \dot{\kappa} \mathbf{p}_{a,\kappa} = \{\mathbf{p}_a, h_J\} = -\frac{\delta H_J}{\delta \mathbf{u}_a}, \quad (3.20)$$

where (3.18) and then (3.9) have been used step by step. Therefore, \mathbf{u}_a is the solution of the sourceless Lagrange equation (2.26) and $\mathbf{p}_a = \dot{\mathbf{u}}_a$.

We will consider the following solution of (3.9):

$$h_J(\lambda, \kappa) = H_J(\mathbf{u}, \mathbf{p}), \quad (3.21)$$

i.e. the case where h is the transformed Hamiltonian and

$$\mathbf{Y}_a(\mathbf{x}; \lambda, \kappa) = \mathbf{u}_a(\mathbf{x}; \lambda, \kappa). \quad (3.22)$$

4 Reduction

4.1 Cyclic variables

Proposition 3 means that α in (3.1) is the coset space index. Let us divide the set $\{\lambda, \kappa\}$ into two parts:

$$\{\lambda, \kappa\} \rightarrow (\{\lambda, \kappa\}, \{\xi', \eta'\}), \quad (4.1)$$

assuming that λ and κ are cyclic variables:

$$\frac{\partial \mathbf{u}_a}{\partial \lambda} = O(\epsilon), \quad \frac{\partial \mathbf{u}_a}{\partial \kappa} = O(\epsilon), \quad \epsilon \rightarrow 0, \quad (4.2)$$

and the derivatives of \mathbf{u}_a over ξ' and η' are finite at $\epsilon = 0$. It can be shown that the variables (λ, κ) stay cyclic in the quantum sense as well.

Proposition 4. *The quantum force is orthogonal to the cyclic variables axes.*
Indeed, taking into account (4.2),

$$\mathbf{k}(j, e) = \int dt \left\{ \frac{\delta}{\delta j_\lambda} \cdot \frac{\delta}{\delta e_\lambda} + \frac{\delta}{\delta j_\kappa} \cdot \frac{\delta}{\delta e_\kappa} + \frac{\delta}{\delta j_{\xi'}} \cdot \frac{\delta}{\delta e_{\xi'}} + \frac{\delta}{\delta j_{\eta'}} \cdot \frac{\delta}{\delta e_{\eta'}} \right\}. \quad (4.3)$$

As it follows from (3.18),

$$\frac{\delta \mathbf{u}_a}{\delta j_X} \sim \frac{\delta \mathbf{u}_a}{\delta X} = O(\epsilon), \quad X = (\lambda, \kappa). \quad (4.4)$$

Therefore, we can write in the limit $\epsilon = 0$ that

$$2\mathbf{k}(j, e) = \int dt \left\{ \frac{\delta}{\delta j_{\xi'}} \cdot \frac{\delta}{\delta e_{\xi'}} + \frac{\delta}{\delta j_{\eta'}} \cdot \frac{\delta}{\delta e_{\eta'}} \right\}. \quad (4.5)$$

Then, following our definition, one should take everywhere

$$j_X = e_X = 0, \quad X = (\lambda, \kappa). \quad (4.6)$$

Q.E.D.

The result of the reduction looks as follows:

$$DM(u, p) = d\Omega \, DM(\xi', \eta'), \quad (4.7)$$

where the infinite dimensional integral over

$$d\Omega = \prod_{\alpha, t} d\lambda_\alpha(t) d\kappa_\alpha(t) \delta(\dot{\lambda}_\alpha(t)) \delta(\dot{\kappa}_\alpha(t)) \quad (4.8)$$

will be cancelled by normalization. This procedure completes the renormalization of the transformed formalism.

The remaining degrees of freedom are entered into the reduced Dirac measure:

$$DM(\xi', \eta') = \prod_t d\xi'(t) d\eta'(t) \delta(\dot{\xi}' - h_{\eta'}(\xi', \eta') - j_{\xi'}) \delta(\dot{\eta}' + h_{\xi'}(\xi', \eta') + j_{\eta'}). \quad (4.9)$$

This result presents the first step of the reduction into the physical coset space W .

4.2 Quantization rule in the coset space

The case when only the part of variables are cyclic: $\{\xi'\} = (\{\xi\}, \{\xi''\})$ and $\{\eta'\} = (\{\eta\}, \{\eta''\})$,

$$\dim\{\xi\} = \dim\{\eta\}, \quad (4.10)$$

where only $\{\xi''\}$ is the set of cyclic variables:

$$\frac{\partial \mathbf{u}_a}{\partial \xi''} = \epsilon, \quad (4.11)$$

comes within the conditions of Proposition 4.

Then we can define the set $\{\eta\}$ under the condition: $\partial h / \partial \eta \neq 0$. In the frame of this definition η_α are the integrals of motion. This gives:

$$DM(\xi, \eta'', \eta) = \prod_t d\eta''(t) d\eta'(t) d\xi(t) \delta(\dot{\eta}'' - j_{\eta''}) \delta(\dot{\eta} + j_\eta) \delta(\dot{\xi} - h_\eta(\eta) - j_\xi). \quad (4.12)$$

Following (3.19) and (3.22) the virtual deviation \mathbf{e} looks as follows:

$$\mathbf{e}_a = e_\xi \mathbf{u}_{a\xi} + e_\eta \mathbf{u}_{a\eta} + e_{\eta''} \mathbf{u}_{a\xi''} \quad (4.13)$$

and the perturbations generating operator is:

$$2\mathbf{k}(j, e) = \int dt \left\{ \frac{\delta}{\delta j_\xi} \frac{\delta}{\delta e_\xi} + \frac{\delta}{\delta j_\eta} \frac{\delta}{\delta e_\eta} + \frac{\delta}{\delta j_{\eta''}} \frac{\delta}{\delta e_{\eta''}} \right\}. \quad (4.14)$$

As it follows from the general condition that the auxiliary variables must be taken equal to zero, we must put $e_{\eta''} = 0$ since (4.11). We must omit simultaneously the last term in (4.14). For this reason one must put $j_{\eta''} = 0$ in (4.12).

Proposition 5. *The coset space quantized variables form the even dimensional symplectic manifold, $\{\xi, \eta\} \in T^*W$, with the canonical equal-time Poisson brackets:*

$$\{\mathbf{u}(\mathbf{x}; \xi, \eta), \mathbf{u}(\mathbf{y}; \xi, \eta)\} = \{\mathbf{p}(\mathbf{x}; \xi, \eta), \mathbf{p}(\mathbf{y}; \xi, \eta)\} = 0, \quad \{\mathbf{u}(\mathbf{x}; \xi, \eta), \mathbf{p}(\mathbf{y}; \xi, \eta)\} = \delta_{\mathbf{x}, \mathbf{y}} \quad (4.15)$$

iff $\{\mathbf{x}\} \not\subseteq \{\alpha\}$. One must insert (3.21) into (3.9) in order to prove this proposition.

Proposition 5 means that $\{\Lambda_a\} \not\subseteq \{\xi, \eta\}$.

4.3 Concluding expression

As a result,

$$\mathcal{N} = e^{-i\mathbf{k}(je)} \int DM(\xi, \eta) e^{iU(\mathbf{u}, \mathbf{e})}, \quad (4.16)$$

where the new coset space virtual deviation is

$$\mathbf{e}_a = \sum_{\alpha} \left\{ e_{\xi_{\alpha}} \frac{\partial \mathbf{u}_a}{\partial \eta_{\alpha}} + e_{\eta_{\alpha}} \frac{\partial \mathbf{u}_a}{\partial \xi_{\alpha}} \right\}. \quad (4.17)$$

The generating quantum perturbations operator in the coset space is

$$2\mathbf{k}(je) = \sum_{\alpha} \int dt \left\{ \frac{\delta}{\delta j_{\xi_{\alpha}}(t)} \frac{\delta}{\delta e_{\xi_{\alpha}}(t)} + \frac{\delta}{\delta j_{\eta_{\alpha}}(t)} \frac{\delta}{\delta e_{\eta_{\alpha}}(t)} \right\}, \quad (4.18)$$

where summation is performed over all canonical pairs, $(\xi, \eta) \in T^*W$. The corresponding measure

$$DM = dR \prod_{\alpha, t} d\xi_{\alpha}(t) d\eta_{\alpha}(t) \delta(\dot{\xi}_{\alpha} - h_{\eta_{\alpha}}(\eta) - j_{\xi_{\alpha}}) \delta(\dot{\eta}_{\alpha} + j_{\eta_{\alpha}}), \quad (4.19)$$

where dR is the zero modes Cauchy measure:

$$dR = \prod_{\alpha, t} d\eta''_{\alpha} \delta(\dot{\eta}''_{\alpha}). \quad (4.20)$$

Therefore,

$$W = T^*W + R \quad (4.21)$$

where $\{\xi, \eta\} \in T^*W$ and $\{\xi''\} \in R$.

The coset space Hamiltonian equations:

$$\dot{\xi}_{\alpha} - h_{\eta_{\alpha}}(\eta) = j_{\xi_{\alpha}}, \quad \dot{\eta}_{\alpha} = -j_{\eta_{\alpha}} \quad (4.22)$$

are easily solved through the Green function $g(t-t')$. The latter must obey the equation:

$$\partial_t g(t-t') = \delta(t-t'). \quad (4.23)$$

This Green function has the universal meaning, and it must be the same for the arbitrary theory. Then, using the $i\varepsilon$ -prescription and the experience of the Coulomb problem considered in [22], we will use the following solution of (4.23):

$$g(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}. \quad (4.24)$$

The solution of the Eq.(4.22) looks as follows:

$$\xi_a^j(t) = \int dt' g(t-t') \{h_{\eta_{\alpha}}(\eta^j) + j_{\xi_{\alpha}}\}(t'), \quad \eta_a^j(t) = - \int dt' g(t-t') j_{\eta_{\alpha}}(t'). \quad (4.25)$$

As a result, the functional measure DM is reduced to the Cauchy measure

$$dM = \prod_{\alpha, t} d\eta''_{\alpha}(t) \delta(\dot{\eta}''_{\alpha}) d\xi_{\alpha}(t) d\eta_{\alpha}(t) \delta(\dot{\xi}_{\alpha}) \delta(\dot{\eta}_{\alpha}) = \prod_{\alpha} d\eta''_{\alpha}(0) d\xi_{\alpha}(0) d\eta_{\alpha}(0). \quad (4.26)$$

The integral over dM gives the volume V of the factor group \mathcal{G}/\mathcal{H} and $\ln V \leq \dim W$. Notice that the gauge group volume V_Λ in our formalism is defined by the measure $\prod_{a,\mathbf{x}} d\Lambda_a(\mathbf{x}, 0)$.

Therefore,

$$\mathcal{N} = e^{-i\mathbf{k}(je)} \int dM e^{iU(\mathbf{u}^j, \mathbf{e}^j)}, \quad (4. 27)$$

where \mathbf{u}^j and \mathbf{e}^j are dependents on (ξ_a^j, η_a^j) functions.

4.4 Gauge invariance

Following (2. 18) and (4. 17),

$$\begin{aligned} U(u, e) &= S(u + e) - S^*(u - e) - 2\text{Re} \sum_{a,\alpha} \int_C dx \left\{ e_{\xi_\alpha} \frac{\partial u_{ai}(x)}{\partial \eta_\alpha} + e_{\eta_\alpha} \frac{\partial u_{ai}(x)}{\partial \xi_\alpha} \right\} \frac{\delta S(u)}{\delta u_{ai}(x)} = \\ &= S(u + e) - S^*(u - e) - 2\text{Re} \sum_\alpha \int_C dt \left\{ e_{\xi_\alpha}(t) \frac{\delta}{\delta \eta_\alpha(t)} + e_{\eta_\alpha}(t) \frac{\delta}{\delta \xi_\alpha(t)} \right\} S(u). \end{aligned} \quad (4. 28)$$

This quantity is transparently gauge invariant.

We can conclude that each term of the coset space perturbation theory is gauge invariant since DM in (4. 19) and $\mathbf{k}(je)$ in (4. 18) are the gauge invariant quantities.

It is interesting to note that in spite of the fact that each term of the perturbation theory is transparently gauge invariant, nevertheless, one can not formulate the theory in terms of the gauge field strength.

5 Conclusions

It is useful to summarize the rules of the coset space perturbation theory.

(i) The transformation, (3. 1), to independent variables, (3. 7), is performed having in mind that the power of the variables set should not be altered, see (3. 2).

(ii) The "host free" transformation is induced by the function \mathbf{u}_a defined by the Eq.(3. 9). In this stage the function $h_j(\lambda, \kappa)$ is arbitrary.

(iii) If h_J is chosen as the liner function of \mathbf{J}_a , $h_J = h + O(J)$ then there exists a mapping into the (λ, κ) space, see (3. 14). This mapping produces a new set of sources $\{j_\lambda, j_\kappa\}$, (3. 16), and virtual deviations, $\{e_\lambda, e_\kappa\}$, (3. 19). It is remarkable that each degree of freedom of the (λ, κ) space is excited independently of one another by the individual sources $\{j_\lambda, j_\kappa\}$. This is crucial for the reduction of the quantum degrees of freedom.

(iv) One can consider the case when a subset of variables is cyclic, (4. 1), (4. 2). The latter means that if $h = h + O(\epsilon)$, $\epsilon \rightarrow 0$, then because of the perturbation term, $O(\epsilon)$, \mathbf{u}_a occupies the whole infinite dimensional space Γ_∞ . Within the limit of $\epsilon = 0$ the trajectory subsides on the surface, W , of a smaller dimension. The redundant variables at $\epsilon = 0$ are cyclic. As a result we have found the reduced measure (4. 9), and the perturbations generating operator (4. 5). The volume of the cyclic variables, (4. 8), is cancelled by normalization. The field theoretical problem becomes finite dimensional.

The cancellation of the cyclic variables volume can be considered as a renormalization procedure.

(v) One may consider now the case when h_J is the transformed Hamiltonian. For this reason W is the coset space. The physical value of $\dim W$ is defined by Corollary. In other respects the choice of the coset variables $\{\xi, \eta\}$ is arbitrary.

(vi) A portion of the remaining variables can belong to the symplectic subspace $T^*W \subseteq W$, with the Poisson brackets (4. 15). The latter allows to conclude that the gauge phase Λ_a can not belong to T^*W . As a result the perturbation theory is transparently gauge invariant.

(vii) The known solution [3] shows that all space-time integrals of the coset space perturbation theory are finite outside the border ∂W since $|S(u)| < \infty$ and $\dim W$ is finite. The border contributions, $\sup(\xi, \eta) \in \partial W$, remain finite because of the $i\varepsilon$ -prescription. Further analysis of the role of the border singularities, see also [22], will be given in subsequent publications.

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